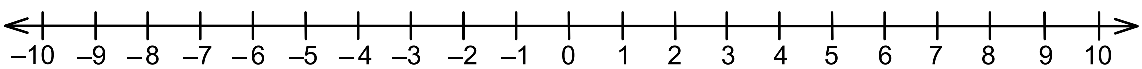
Chapter 1 – Operations With Numbers

Part I – Negative Numbers and Absolute Value

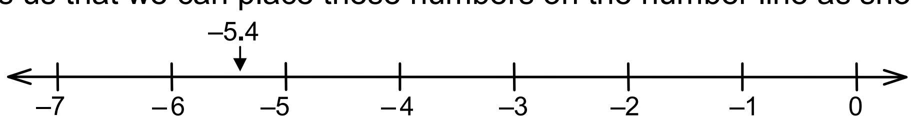
You may already know what negative numbers are, but even if you don't, then you have probably seen them several times over the past few days. If you look at a thermometer, for instance, there are numbers with a minus sign in front of them. Sometimes, football game statistics have numbers with a minus sign in front of them. These numbers are called negative numbers, and they are numbers that are less than zero. The number line below shows some of these numbers, and the arrows on each end of the line tell us that the line continues forever in both directions. In other words, there is no smallest number or largest number in the world.



Often, as in the case of football game statistics, negative numbers refer to a loss of something. For example, if you see a statistics that says "Passing yards: –21," then you would know that, when they tried to pass the ball, they were pushed back 21 yards instead of moving forward. As another example, electric bills sometimes have negative numbers on the line that tells you how much money you owe them. This means that, instead of you owing them money, they owe you that much money. Finally, the numbers that you are used to working with are called positive numbers. You can put a plus sign in front of these numbers if you want to, but you do not have to.

Example 1: List the numbers 0, –5, –6, and –5.4 in order from smallest to largest.

We will begin by noting that we can think of the number –5 as 5 units below zero, and we can think of –6 as 6 units below zero. Next, we will note that the number +5.4 is between +5 and +6 (and a little bit closer to the +5), and so, since –5.4 is 5.4 below zero, –5.4 must be between –5 and –6 (and a little bit closer to the –5). This tells us that we can place these numbers on the number line as shown below.



This tells us that the list –6, –5.4, –5, and 0 correctly lists these numbers in order from smallest to largest.

Now, we need a definition.

The absolute value of a number is defined as its distance from zero.

In other words, to find the absolute value of a number, just make the number positive (or keep it zero if it is zero to start out with).

We talk about the absolute value of a number by putting the number inside two vertical lines, as shown in the following examples.

|–2| = the absolute value of –2 = 2 (since –2 is 2 units from 0)
|3| = the absolute value of 3 = 3 (since +3 is 3 units from 0)
|0| = the absolute value of 0 = 0 (since 0 is 0 units from 0)

Example 2: Simplify |15| + |–7|.

We will begin by stating that, when a question tells you to *simplify* an expression, the question is asking you to write an equivalent expression that looks as uncomplicated as possible.

Now, since |15| = 15 and |–7| = 7, we can say that |15| + |–7| = 15 + 7. Since this simplifies further to 22, we can say that |15| + |–7| = 22.

Before we look at the next example, we will state that we sometimes want to talk about values that change, or vary. When we do this, we often use <u>variables</u>, or letters or symbols that represent a quantity that can change. For example, we can say that x + 3 represents some number plus 3. As another example, suppose a rental car company charges you $25 for each day you want to rent a car. Then, to talk about how much the company will charge you, we can say c = 25 × n, where c represents the cost of renting the car and n represents the number of days you want to rent the car. We will talk more about this subject in the next chapter.

In this book, we will not use the symbol "×" to mean multiply because the letter x is the most commonly used variable, and we want to avoid confusion. Instead, we will use a dot, parentheses, or, if a variable is multiplied by a number or another variable, nothing at all. The following examples illustrate this concept.

3 • 4 = 3 times 4
3(4) = 3 times 4
3 • 4 • 2 = 3 times 4 times 2
3(4)(2) = 3 times 4 times 2
3y = 3 times y
3xy = 3 times x times y
34 = thirty-four

Notice that, in each of the above examples, if a number is multiplied by one or more variables, then the variable(s) are listed *after* the numbers. Also, the variables are listed in alphabetical order. These are conventions that will be used, for the most part, throughout the rest of the book.

Now, we can look at another example.

Example 3: Simplify 4|–9| – |–3|.

We can work this problem by following the steps shown below. The statements in gray beside each step explain what we did in that step.

<u>Steps</u>	<u>Explanations of Steps</u>
4|–9| – |–3| = 4 • 9 – 3	Note that |–9| = +9 and that |–3| = +3. Also note that, since there is no sign between the 4 and the first set of absolute value symbols, we must assume multiplication.
= 36 – 3	Note that 4 • 9 = 36.
= 33	

Problems:

1. Write the number represented: a loss of $35 ____________

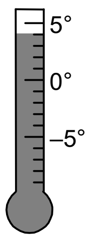
2. Write the number represented: a gain of $21.34 ____________

3. What is one way of interpreting the number "+15"? ____________________________

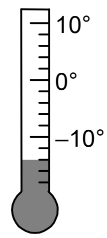
4. What is one way of interpreting the number "–12"? ____________________________

5-8. Write the temperature represented by each of the following thermometers.

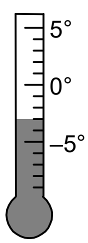
5. ____________



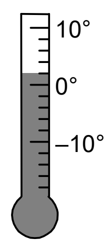
7. ____________



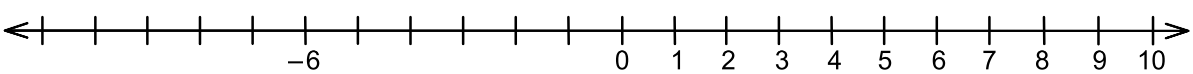
6. ____________



8. ____________



9. Fill in the missing numbers.



10. Suppose that a football player gains 20 yards and then loses 3 yards. Write a number sentence to describe this situation and tell what the end result is.

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11. Suppose that a football player loses 6 yards on each of 3 plays. Write a number sentence to describe this situation and tell what the end result is.

____

12. To talk about how high a mountain is or how deep a valley is, we often give the elevation relative to the average height of the surface of the oceans, or *sea level*. We say that sea level has an elevation of 0 feet. Suppose that a person goes to the top of a mountain that is 8,000 feet above sea level and then descends 500 feet. Write a number sentence to describe this situation and tell what the end result is.

____

13. Suppose that a person purchases a shirt for $25, then returns a shirt for $18, and then buys a sandwich for $2. Write a number sentence to describe his cash flow, and tell what the end result is. ____

14-17. List each of the following sets of numbers in order from smallest to largest.

14. 6, –7, –5, 0, –7¾ ___________________________

15. –1, –⅝, 0, –5, 8 ___________________________

16. 0, –9.8, –8.23, –8, 2, –9 ___________________________

17. –5.9, –3.5, –4, –6, –8 ___________________________

18-26. Simplify each of the following.

18. |–5| = ___________

19. 4|12| = ___________

20. |–18| ÷ |3| = ___________

21. |15| – |–8| = ___________

22. 3|–11| = ___________

23. 7|3| + |–2| = ___________

24. 5|–7| – |–2| = ___________

25. 12 ÷ |–3| = ___________

26. 6|4| – |5| = ___________

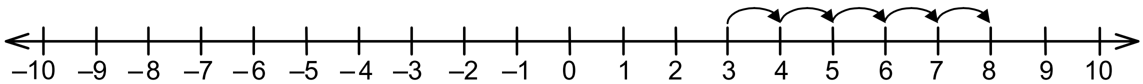
Part II – Operations With Positive and Negative Numbers

In this section, we will discuss how we add, subtract, multiply, and divide positive and negative numbers.

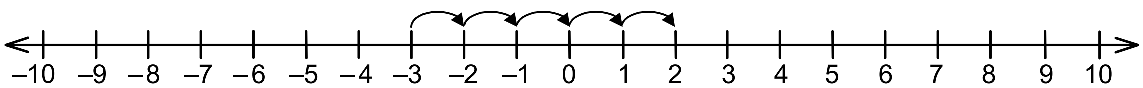
Example 1: Simplify each of the following: (a) 3 + 5, (b) –3 + 5, (c) 3 + (–5), and (d) –3 + (–5).

To answer all of these problems, we will look at the number line.

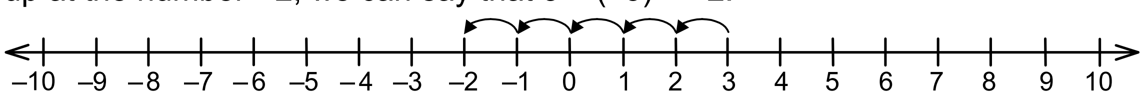
(a) To find the answer to this problem, we start at the number 3 and then move to the right 5 units. Since we wind up at the number 8, we can say that 3 + 5 = 8.



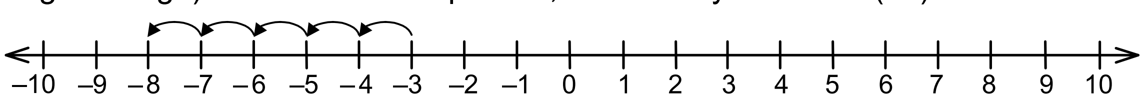
(b) To find the answer to this problem, we start at the number –3 and then move 5 units to the right. Since we end up at 2, we can say that –3 + 5 = 2.



(c) Many students find the parentheses in this type of problem confusing. Because there is an addition sign between our numbers, the parentheses in this problem are not doing anything except making the problem easier to look at. Thus, when we write 3 + (–5), it is exactly the same as writing 3 + –5. To find the answer to this problem, we start at the number 3 and then, because we want to add a *negative* 5, we must move to the *left* 5 units. Since we end up at the number –2, we can say that 3 + (–5) = –2.



(d) Finally, let's consider the number sentence –3 + (–5). We start at the number –3 and then move to the left 5 units (again, we move to the left because of the negative sign). Since we end up at –8, we can say that –3 + (–5) = –8.



If you think about these examples and the mathematics behind them, you can come up with the following rules for adding two signed numbers: (1) If the signs are both positive or both negative, add the absolute values of the numbers and keep the same sign, and (2) If you want to add one positive number and one negative number, subtract the absolute values of the numbers and keep the sign of the number that has the larger absolute value. Study the following examples.

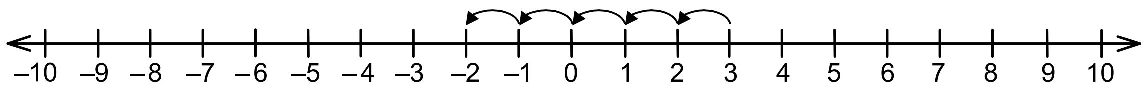
–5 + (–1) = –6	58 + 84 = 142
5 + 1 = 6	–58 + (–84) = –142
–5 + 1 = –4	58 + (–84) = –26
5 + (–1) = 4	–58 + 84 = 26
5 + (–5) = 0	–58 + 58 = 0

Now, let's talk about how to subtract positive and negative numbers. In general, minus and negative signs tell you to reverse your direction.

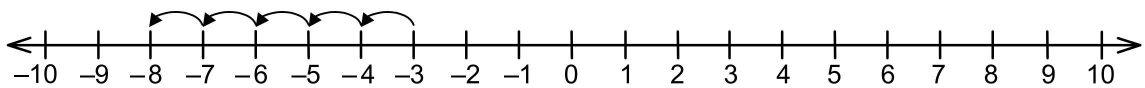
Example 2: Simplify (a) $3 - 5$, (b) $-3 - 5$, (c) $3 - (-5)$, and (d) $-3 - (-5)$.

To work these examples, we will look at the number line again.

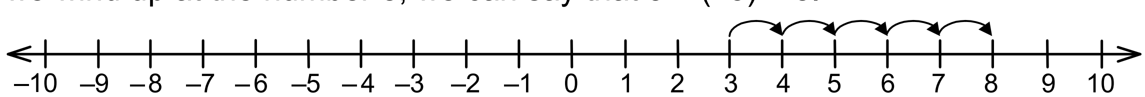
(a) To find out what $3 - 5$ equals, we start at the number 3, and then we must move to the left (because the minus sign tells us to reverse the direction) 5 units. We wind up at the number -2, and so we can say that $3 - 5 = -2$.



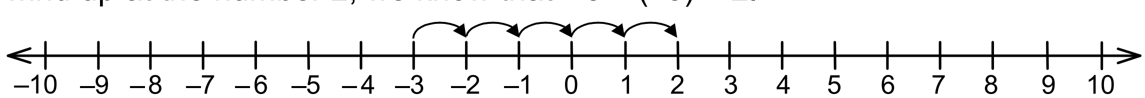
(b) Now, let's consider the number sentence $-3 - 5$. This time, we start at the number -3, and then we find out we must move to the left (because the minus sign tells us to reverse the direction) 5 units. We end up at the number -8, and so we know that $-3 - 5$ is equal to -8.



(c) Next, we will look at a trickier example: $3 - (-5)$. We start out at 3 again. The subtraction sign tells us to move to the left, but then, before we find out how many units we should move, we see that we must reverse our direction again and move to the right. We see that we must move to the right 5 units. Since we wind up at the number 8, we can say that $3 - (-5) = 8$.



(d) To simplify $-3 - (-5)$, we start out at -3. The subtraction sign tells us to move to the left, but then, before we find out how many units to move, we see that we must reverse our direction again and move to the right 5 units. Since we wind up at the number 2, we know that $-3 - (-5) = 2$.



These examples and the mathematics behind them give rise to the following rule for subtracting positive and negative numbers: Change the subtraction sign to addition and change the sign of the next number. You then have an addition problem, and so you can then follow the rules for addition that were discussed earlier. To see why this rule works, note that we did exactly the same thing when we worked part (c) of Example 1 and part (a) of Example 2, and so we can conclude that $3 - 5$ must equal $3 + (-5)$. The following examples illustrate this rule further.

$7 - 13 = 7 + (-13) = -6$

$-7 - 13 = -7 + (-13) = -20$

$7 - (-13) = 7 + (+13) = 20$

$-7 - (-13) = -7 + (+13) = 6$

$7 - 7 = 7 + (-7) = 0$

$32 - 14 = 32 + (-14) = 18$

$-32 - 14 = -32 + (-14) = -46$

$32 - (-14) = 32 + (+14) = 46$

$-32 - (-14) = -32 + (+14) = -18$

$-32 - (-32) = -32 + (+32) = 0$

Now, let's look at how to multiply positive and negative numbers.

Example 3: Simplify (a) $2 \cdot 4$, (b) $-2 \cdot 4$, (c) $2 \cdot (-4)$, and (d) $(-2)(-4)$.

(a) You can think of this as gaining \$2 four times (or, equivalently, gaining \$4 twice). This means you gained \$8, and so we can say that $2 \cdot 4 = 8$.

(b) You can think of $-2 \cdot 4$ as losing \$2 four times. This would mean that you lost a total of \$8. Thus, we can say that $-2 \cdot 4 = -8$.

(c) You can think of $2 \cdot (-4)$ as losing \$4 twice. This would mean that you lost \$8, and so we can say that $2 \cdot (-4) = -8$.

(d) Finally, we will look at how to multiply $(-2)(-4)$. As it turns out, $(-2)(-4) = +8$. To see why, look at the patterns shown below.

$3(-4) = -12$	$-2(1) = -2$
$2(-4) = -8$	$-2(0) = 0$
$1(-4) = -4$	$-2(-1) = 2$
$0(-4) = 0$	$-2(-2) = 4$
$-1(-4) = 4$	$-2(-3) = 6$
$-2(-4) = 8$	$-2(-4) = 8$

These examples give rise to the following two rules: (1) When you multiply two numbers with the same sign, the answer will be positive, and (2) When you multiply two numbers with different signs, the answer will be negative. Study the following examples.

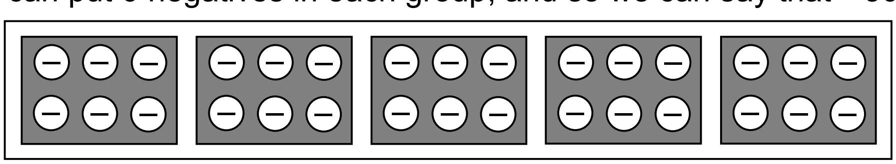
$8(3) = 24$	$(5)(-2) = -10$
$-8(3) = -24$	$(-5)(-2) = 10$

Finally, we will look at how to divide positive and negative numbers.

Example 4: Simplify (a) $30 \div 5$, (b) $-30 \div 5$, (c) $30 \div (-5)$, and (d) $-30 \div (-5)$.

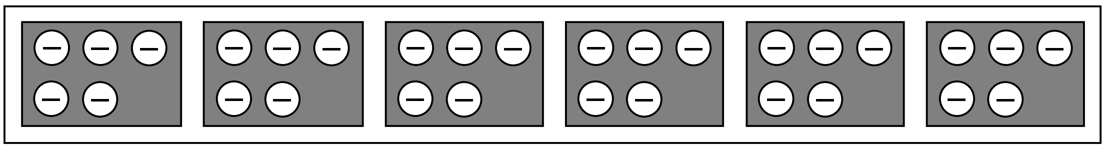
(a) We can look at this problem in three different ways. (1) We can say that this problem wants us to take 30 items and split them into 5 equal groups. Since there would be 6 items in each group, we can say that $30 \div 5 = 6$. (2) We can say that this problem wants us to take 30 items and split them into equal groups with 5 items in each group. Since there would be 6 groups, we can say that $30 \div 5 = 6$. (3) We can say that we want to find the number that gives us 30 when it is multiplied by 5. Since $(6)(5) = 30$, we can say that $30 \div 5 = 6$.

(b) Next, let's look at $-30 \div 5$. To find out what this equals, you can ask yourself, "What can I multiply by 5 to get -30?" Since the answer to this question is -6, we can say that $-30 \div 5 = -6$. Or, if you prefer, you can ask yourself, "How can I take -30 and split it into 5 equal groups?" As the picture below shows, you can put 6 negatives in each group, and so we can say that $-30 \div 5 = -6$.



(c) Next, let's look at 30 ÷ (–5). It turns out that 30 ÷ (–5) = –6, and the easiest way to understand why is to realize that that the question is telling us to ask ourselves, "What can I multiply by –5 to get 30?"

(d) Finally, let's look at –30 ÷ (–5). We can figure out what this equals by asking ourselves, "What can I multiply by –5 to get –30?" Since 6(–5) = –30, we can say that –30 ÷ (–5) = 6. We can also ask ourselves, "How can I split –30 up into equal groups so that there are 5 negatives in each group?" As the picture below shows, we can do this by making 6 groups, and so we can say that –30 ÷ (–5) = 6.



These examples tell us that the rules for division are exactly the same as the rules for multiplication: (1) When you divide two numbers with the same sign, the answer will be positive, and (2) When you divide two numbers with different signs, the answer will be negative. Study the examples below.

18 ÷ 3 = 6 –40 ÷ 8 = –5

18 ÷ (–3) = –6 –40 ÷ (–8) = 5

The following chart summarizes the rules for adding and subtracting positive and negative numbers. The words across the top indicate the sign in the middle.

	Addition	Subtraction	Multiplication or Division
2 positives or 2 negatives (like signs)	Add the absolute values of the numbers and keep the same sign.	Change the subtraction sign to addition and change the sign of the next number.	Multiply or divide the absolute values of the numbers. The sign is positive.
1 positive and 1 negative (unlike signs)	Subtract the absolute values of the numbers and keep the sign of the number with the larger absolute value.		Multiply or divide the absolute values of the numbers. The sign is negative.

Problems – Simplify each of the following.

27. –16 + 4 = ________________

29. 3(4) = ________________

28. 4 – (–9) = ________________

30. 5 + (–21) = ________________

31. 64 – 59 = ________________

32. 14 ÷ (–2) = ________________

33. –31 – 6 = ________________

34. –28 ÷ 7 = ________________

35. –8 – (–17) = ________________

36. (–8)(–4) = ________________

37. 6 + 19 = ________________

38. –10 + 6 = ________________

39. –9(–7) = ________________

40. 48 – 9 = ________________

41. 14 ÷ 7 = ________________

42. –5(–4) = ________________

43. 7 – 29 = ________________

44. –18 + (–6) = ________________

45. –27 ÷ (–9) = ________________

46. 25 ÷ (–5) = ________________

47. 19 + (–2) = ________________

48. 4(6) = ________________

49. –5 • 7 = ________________

50. 81 – (–5) = ________________

51. 12 ÷ (–3) = ________________

52. –7(–7) = ________________

53. –17 + 8 = ________________

54. –15 + (–17) = ________________

55. –32 ÷ 8 = ________________

56. –8 – (–15) = ________________

Part III – Review of Operations With Fractions

If you get confused on the rules we follow when working with fractions, the chart below should help you.

Reduce	Change an improper fraction to a mixed number	Change a mixed number to an improper fraction	Add or subtract with like denominators (bottom numbers)	Add or subtract with unlike denominators	Multiply	Divide
Divide both the bottom number and the top number by the biggest thing you can. Then, repeat if you can.	Divide the top number by the bottom number. The answer is the whole number, and the remainder is the top number. The bottom number stays the same.	Multiply the whole number by the bottom number, and add this answer to the top number. The bottom number stays the same.	Change all the mixed numbers and whole numbers to improper fractions.* Then add or subtract the top numbers (or numerators) and keep the bottom numbers the same.	Change all the mixed numbers and whole numbers to improper fractions.* Next, you need to get a common denominator. Do this by finding the least common multiple of the bottom numbers. Change each of the fractions into equivalent fractions with this new common denominator by multiplying the top and bottom by the same number. Then, add or subtract the top numbers and keep the bottom the same.	Change all the mixed numbers and whole numbers to improper fractions. Then multiply straight across.	Change all the mixed numbers and whole numbers to improper fractions. Then copy down the first fraction, change the division sign to multiplication, and flip the second fraction.
Example 1: Reduce $\frac{16}{48}$. $\frac{16 \div 16}{48 \div 16} = \frac{1}{3}$ or, equivalently, $\frac{16 \div 2}{48 \div 2} = \frac{8 \div 8}{24 \div 8}$ Either way, $\frac{16}{48} = \frac{1}{3}$.	Example 2: Change $\frac{65}{9}$ to a mixed number. $65 \div 9 = 7$ remainder 2 $\frac{65}{9} = 7\frac{2}{9}$	Example 3: Change $3\frac{5}{12}$ to an improper fraction. $3 \cdot 12 = 36$ $36 + 5 = 41$ $3\frac{5}{12} = \frac{41}{12}$	Example 4: Simplify $\frac{3}{5} + \frac{1}{5}$. $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$	Example 5: Simplify $\frac{3}{5} + \frac{2}{7}$. LCM of 5 and 7 = 35 $\frac{3}{5} = \frac{?}{35}$ and $\frac{2}{7} = \frac{?}{35}$ $\frac{3 \cdot 7}{5 \cdot 7} = \frac{21}{35}$ $\frac{2 \cdot 5}{7 \cdot 5} = \frac{10}{35}$ $\frac{3}{5} + \frac{2}{7} = \frac{21}{35} + \frac{10}{35} = \frac{31}{35}$	Example 6: Simplify $\frac{5}{8} \bullet \frac{12}{15}$. $\frac{5}{8} \bullet \frac{12}{15} = \frac{60}{120}$ $\frac{60 \div 60}{120 \div 60} = \frac{1}{2}$	Example 7: Simplify $\frac{4}{9} \div \frac{5}{6}$. $\frac{4}{9} \div \frac{5}{6} = \frac{4}{9} \bullet \frac{6}{5}$ $\frac{4}{9} \bullet \frac{6}{5} = \frac{24}{45}$ $\frac{24 \div 3}{45 \div 3} = \frac{8}{15}$

*When you are adding and subtracting fractions, you can add or subtract the fractions and whole numbers separately instead of changing the mixed numbers and whole numbers to improper fractions first. However, this often involves processes called *borrowing* and/or *renaming*. Many students find these processes confusing, but, if you understand them, you are welcome to use these methods instead. You will get the same answer in the end.

Now, let's look at how we can apply our rules for operations with positive and negative numbers to fractions.

Example 8: Simplify $-2\frac{5}{6}-\left(-\frac{9}{10}\right)$.

We can work this problem by following the steps shown below.

$-2\frac{5}{6}-\left(-\frac{9}{10}\right) = -2\frac{5}{6}+\frac{9}{10}$ — We will start by changing the subtraction sign to addition and changing the sign of the next number.

$= -\frac{17}{6}+\frac{9}{10}$ — Next, we will change the mixed number into an improper fraction. Note that $-2\frac{5}{6} = -\left(2\frac{5}{6}\right) = -\frac{17}{6}$.

$= -\frac{85}{30}+\frac{27}{30}$ — To add two fractions, we must have a common denominator. The least common multiple of 6 and 10 is 30, and so we must change both our fractions into equivalent fractions with denominators of 30. Note that $-\frac{17}{6} = -\frac{17 \bullet 5}{6 \bullet 5} = -\frac{85}{30}$ and that $\frac{9}{10} = \frac{9 \bullet 3}{10 \bullet 3} = \frac{27}{30}$.

$= -\frac{58}{30}$ — Now, we can add the fractions together. Note that –85 + 27 = –58.

$= -1\frac{14}{15}$ — We must always simplify our answers as much as possible. Note that $-\frac{58}{30}$ reduces to $-\frac{29}{14}$, which simplifies further to $-1\frac{14}{15}$. (You could also change $-\frac{58}{30}$ to a mixed number and then reduce the answer. The final answer would be the same.)

Example 9: Simplify $-3\frac{5}{7}-4\frac{1}{2}$.

We can work this problem by following the steps shown below.

$-3\frac{5}{7}-4\frac{1}{2} = -\frac{26}{7}-\frac{9}{2}$ — Note that $-3\frac{5}{7} = -\left(3\frac{5}{7}\right) = -\frac{26}{7}$ and that $4\frac{1}{2} = \frac{9}{2}$.

$= -\frac{52}{14}-\frac{63}{14}$ — To add or subtract fractions, we must have a common denominator. The least common multiple of 7 and 2 is 14, and so we must change both our fractions into equivalent fractions with denominators of 14. Note that $-\frac{26}{7} = -\frac{26 \bullet 2}{7 \bullet 2} = -\frac{52}{14}$ and that $\frac{9}{2} = \frac{9 \bullet 7}{2 \bullet 7} = \frac{63}{14}$.

$= -\frac{115}{14}$ — Now, we can subtract the fractions. Note that –52 – 63 = –52 + (–63), or –115.

$= -8\frac{3}{14}$ — We must always simplify our answers as much as possible. Note that $-\frac{115}{14}$ simplifies to $-8\frac{3}{14}$.

Example 10: Simplify $(-10)\left(-1\frac{7}{8}\right)$.

We will begin by noting that, because we are multiplying two negative numbers in this problem, we must multiply the absolute values of the numbers and then make the answer positive. This tells us that we can work this problem as shown below.

$(-10)\left(-1\frac{7}{8}\right) = \left(-\frac{10}{1}\right)\left(-\frac{15}{8}\right)$

We must begin by changing the whole number and the mixed number to improper fractions. Note that $-10 = -\frac{10}{1}$ and that $-1\frac{7}{8} = -\left(1\frac{7}{8}\right) = -\frac{15}{8}$.

$= \frac{150}{8}$

Now, we can multiply the absolute values of the fractions and then make the answer positive. Since 10 • 15 = 150 and 1 • 8 = 8, we can say that $\left(\frac{10}{1}\right)\left(\frac{15}{8}\right) = \frac{150}{8}$. Thus, $\left(\frac{10}{1}\right)\left(\frac{15}{8}\right) = \frac{150}{8}$.*

$= 18\frac{3}{4}$

We must always simplify our answer as much as possible. Note that $\frac{150}{8}$ reduces to $\frac{75}{4}$, which simplifies further to $18\frac{3}{4}$.

Problems:

57. Reduce: $\frac{24}{36}$ = ________________

58. Change $2\frac{5}{7}$ to an improper fraction. ________________

59. Change to an improper fraction: $5\frac{3}{4}$ = ________________

60. Change $\frac{34}{4}$ to a mixed number, and reduce the result. ________________

61. Change $\frac{15}{7}$ to a mixed number. ________________

62. Reduce: $\frac{12}{15}$ = ________________

*Before you multiply straight across, you can use a process called *canceling* to reduce the fractions. If you understand this process, you may use it to work the problems that involve multiplying fractions; your final answer will be the same. However, we will not discuss it here because many students have a tendency to get this process confused with other processes.

63-90. Simplify each of the following.

63. $\frac{3}{4}\left(\frac{2}{5}\right)$ = ____________________

64. $\frac{3}{10}+\frac{5}{10}$ = ____________________

65. $\frac{7}{8}+\frac{3}{8}$ = ____________________

66. $\frac{4}{5}-\frac{3}{10}$ = ____________________

67. $\left(\frac{5}{6}\right)\left(\frac{9}{10}\right)$ = ____________________

68. $21 \div \frac{5}{6}$ = ____________________

69. $\frac{5}{9}\div\frac{4}{7}$ = ____________________

70. $\left(4\frac{2}{5}\right)\left(3\frac{1}{4}\right)$ = ____________________

71. $6\left(2\frac{3}{5}\right)$ = ____________________

72. $3\frac{2}{5}-1\frac{5}{8}$ = ____________________

73. $\frac{2}{15}\div 1\frac{5}{6}$ = ____________________

74. $\frac{4}{5}+\left(-\frac{3}{5}\right)$ = ____________________

75. $\frac{8}{9}\left(-\frac{3}{4}\right)$ = ____________________

76. $-3\frac{1}{8}\div 4\frac{2}{3}$ = ____________________

77. $\frac{7}{8} \div \left(-3\frac{2}{5}\right)$ = ______________

78. $1\frac{2}{5} - 4\frac{1}{3}$ = ______________

79. $1\frac{7}{10} - \left(-\frac{4}{5}\right)$ = ______________

80. $-2\frac{1}{2} \div \left(-\frac{5}{6}\right)$ = ______________

81. $-\frac{6}{7} - \frac{2}{5}$ = ______________

82. $\frac{5}{9} - \left(-2\frac{2}{3}\right)$ = ______________

83. $-\frac{5}{6} + 1\frac{1}{4}$ = ______________

84. $-5\frac{2}{5}\left(\frac{3}{8}\right)$ = ______________

85. $-3\frac{8}{9} - \left(-\frac{7}{9}\right)$ = ______________

86. $-\frac{4}{9} + \frac{3}{5}$ = ______________

87. $\left(-\frac{3}{5}\right)\left(-\frac{7}{9}\right)$ = ______________

88. $-\frac{5}{8} + \left(-\frac{4}{15}\right)$ = ______________

89. $-\frac{4}{5}\left(-2\frac{7}{8}\right)$ = ______________

90. $-1\frac{6}{7} \div \frac{3}{5}$ = ______________

Part IV – Review of Operations With Decimals

Before we talk about different operations with decimals, we will review two concepts.

- If a decimal is not stated in a number, you should always assume that it goes at the end. Also, you can always put zeros after the decimal as long as you put them after the last number. For example, all of the following numbers are equal to each other.

  14   14.   14.0   14.000000

- In long division, the number inside the division symbol is called the <u>dividend</u>, and the number outside the division symbol is called the <u>divisor</u>.

We can now discuss the primary operations with decimals.

Add or subtract with decimals	Multiply with decimals	Divide with decimals	Convert a fraction to a decimal	Convert a terminating decimal to a fraction
Line up the decimal points, and add zeros after the decimal point and after the last number if you need to. Example 1: Subtract: 126 – 4.23 $\begin{array}{r} \overset{5}{\cancel{6}}.\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\overset{10}{\cancel{0}} \\ 126.00 \\ -\ \ \ 4.23 \\ \hline 121.77 \end{array}$	Ignore the decimal points and multiply the numbers like you normally would. Then count up the total number of digits behind the decimals in the problem. This is the number of digits that should be behind the decimal point in your answer. Example 2: Multiply: (6.24)(0.3) $\begin{array}{r} 6.24 \\ \times\ \ 0.3 \\ \hline 1.872 \end{array}$ (Note that there are a total of 3 digits behind the decimal in the problem and in the answer.)	Bring the decimal in the dividend straight up. However, if the divisor has a decimal in it, you will first need to move it over to the end. When you do this, move the decimal in the dividend over the same number of spaces. Example 3: Divide: 28 ÷ 3.5 We must begin by moving the decimal points as shown below. $3.5\overline{)28.0.}$ Now, we can divide as we normally would. $\begin{array}{r} 8. \\ 35\overline{)280.} \\ -280 \\ \hline 0 \end{array}$	Divide the top number by the bottom number. Add a decimal and zeros onto the end of the dividend if necessary. Example 4: Convert $\frac{3}{20}$ to a decimal. $\begin{array}{r} 0.15 \\ 20\overline{)3.00} \\ -\ 20 \\ \hline 100 \\ -100 \\ \hline 0 \end{array}$ Therefore, $\frac{3}{20} = 0.15$.	Read the number and write what you say as a fraction or as a mixed number. Example 5: Convert 1.63 to a fraction. To read this number correctly, you say, "one and sixty-three hundredths." Therefore, we can say that $1.63 = 1\frac{63}{100}$.

Now, let's look at some examples that show how we can use these rules to perform different operations with positive and negative decimals.

Example 6: Subtract: $9 - 12.4$.

We must begin by noting that we cannot subtract these two numbers without talking about negative numbers because 12.4 is bigger than 9. Therefore, we must recall our rule for subtraction. We must change the subtraction sign to addition and change the sign of the next number. When we do this, our problem becomes $9 + (-12.4)$. Now, since the rule for adding two numbers with different signs says that we must subtract the absolute values of the numbers and keep the sign of the number with the larger absolute value, our final answer is -3.4. (Note that, when you use your pencil and paper to subtract these two numbers, you must do $12.4 - 9$ because you cannot do $9 - 12.4$.)

Example 7: Divide: $-0.08 \div 2$.

First of all, note that the sign in our final answer will be negative because we are dividing two numbers that have different signs. Now, we can divide the absolute values of the numbers.

$$\begin{array}{r} 0.04 \\ 2\overline{)0.08} \\ \underline{-8} \\ 0 \end{array}$$

Thus, our final answer is -0.04.

Example 8: Subtract: $-3\frac{4}{5} - 7.12$.

To work this problem, we will start by changing $-3\frac{4}{5}$ to a decimal. We can do this one of two ways: (1) by noting that $\frac{4}{5} = 0.8$ (using the part of the chart on the previous page that discusses the method we use to change fractions to decimals), and thus $-3\frac{4}{5} = -3.8$, or (2) by noting that $-3\frac{4}{5} = -\frac{19}{5}$, which becomes -3.8 when we change it to a decimal. Either way, our problem becomes $-3.8 - 7.12$. Now, we must remember our rule for subtraction. We must change the subtraction sign to addition and change the sign of the next number. When we do this, our problem becomes $-3.8 + (-7.12)$. Next, since the sign in the middle is now addition and the two numbers have the same sign, we must add the absolute values of the numbers and keep the negative sign. Hence, we can now say that $-3\frac{4}{5} - 7.12 = -10.92$. (By the way, we could also have started by noting that $7.12 = 7\frac{3}{25}$ and then simplifying $-3\frac{4}{5} - 7\frac{3}{25}$. If we had done this, we would have gotten a final answer of $-10\frac{23}{25}$, which is equivalent to -10.92.)

Example 9: Add: $-9.2 + \frac{4}{7}$.

To work this problem, we could change the $\frac{4}{7}$ to a decimal like we did in the last problem. However, when we do this, we find that, as a decimal, $\frac{4}{7}$ is a nasty, ugly number. Therefore, we will change the -9.2 to a mixed number instead. To read this number, we say, “negative nine and two-tenths.” Thus, $-9.2 = -9\frac{2}{10}$ (or, equivalently, $-9\frac{1}{5}$). So, our problem now becomes $-9\frac{1}{5} + \frac{4}{7}$. When you use your rules for adding and subtracting fractions that we discussed earlier in this chapter, you should find that the final answer to this question is $-8\frac{22}{35}$.

Problems – Simplify each of the following.

91. 4.5 + 12 = _______________

92. 620 – 5.71 = _______________

93. 8 – 1.759 = _______________

94. 3.14(7) = _______________

95. 5.41 • 1.2 = _______________

96. (2.7)(5.121) = _______________

97. 5.4 ÷ 3 = _______________

98. 3.4 ÷ 0.4 = _______________

99. 6.8 + (–1.43) = _______________

100. –3.2 + (–54) = _______________

101. –2 – (–3.51) = _______________

102. 5.6 – 82 = _______________

103. $-32 - 5.3 =$ _______________

104. $-8.12 - 63.4 =$ _______________

105. $-3\frac{1}{2} - 4.45 =$ _______________

106. $-6.2(-0.5) =$ _______________

107. $\frac{1}{5}(-1.4) =$ _______________

108. $(-0.4)\left(2\frac{1}{3}\right) =$ _______________

109. $8 \div (-0.5) =$ _______________

110. $-2\frac{1}{6} \div (-0.7) =$ _______________

111. $3.2 - 14 =$ _______________

112. $5.6 \div 1\frac{2}{9} =$ _______________

113. $-3.49 - (-7.5) =$ _______________

114. $\frac{3}{8} - (-0.2) =$ _______________

115. $0.2 \div \left(-2\frac{1}{3}\right) =$ _______________

116. $4 - 7.26 =$ _______________

117. $-2.5 - 4\frac{1}{6} =$ _______________

118. $42 + (-15.2) =$ _______________

Part V – Exponents

Sometimes, when you are working math problems, you will see a little raised number above the rest of the line. These numbers are called <u>exponents</u> or <u>powers</u>. They tell you to multiply the number by itself that many times. For example, let's look at the problem below.

Example 1: Simplify 5^3.

In this case, the 5 is called the <u>base</u>, and the 3 is called the exponent. This problem tells you to multiply 5 by itself 3 times. In other words, it wants you to figure out what 5 • 5 • 5 equals. Therefore, we say $5^3 = 125$.

Now, let's look at how to read these numbers. We read them by first saying the base, then "to the," and then the power. For example, we read "4^{15}" by saying "four to the fifteenth power." Sometimes, however, we replace the words "to the second power" with "squared" and "to the third power" with "cubed." For example, we could read "5^2" as "five to the second power" or "five squared." We could read "5^3" as "five to the third power" or "five cubed."

Before you get started with the problems, there is only one more piece of information you need to know: Anything (except zero) to the zero power will always equal 1. For example, $4^0 = 1$ and $105^0 = 1$.

Problems – Simplify each of the following.

119. 4^2 = ________________

120. 3^4 = ________________

121. 9^2 = ________________

122. 5^1 = ________________

123. 64^0 = ________________

124. 72^0 = ________________

125. 38^1 = ________________

126. 1^6 = ________________

127. $\left(\frac{4}{5}\right)^2$ = ________________

128. $\left(\frac{1}{2}\right)^3$ = ________________

129. Fill in the blank with <, >, or =: 1^8 ___ 1^{15}

130. Fill in the blank with <, >, or =: 5^3 ___ 3^5

Part VI – The Order of Operations

What would you get if you were asked to find out what 3 + 1 • 4 equals? If you are like most people, you probably got 16. You probably added 3 + 1, and then multiplied the answer by 4. Now, try this problem on a calculator. If you have a scientific or graphing calculator, then the calculator told you 7. It multiplied 1 • 4 and then added 3 to that. How do we resolve these conflicting answers?

The answer is that we have a set of rules to follow called the order of operations, which is summarized in the following list. (The phrase "Please Excuse My Dear Aunt Sally" is only a tool to help you remember the order of operations; it has no real relevance to math or to this lesson.)

P	Please	Parentheses
E	Excuse	Exponents
M/D	My Dear	Multiplication and Division in order from left to right
A/S	Aunt Sally	Addition and Subtraction in order from left to right

Scientific calculators follow the order of operations, and that is why they often give an answer that is different from the one you would expect to get when you are working this type of problem. The order of operations says that we must get rid of all parentheses by simplifying any expressions inside any parentheses before we try to do anything else. Then, we must get rid of all the exponents. Next, we can move on to multiplication and division. We must do these operations in order from left to right (like you are reading the words on this page). Finally, we can finish the problem by adding and subtracting in order from left to right (again, like you are reading the words on this page).

Example 1: Simplify $1 - (3 + 7) \div 2 \cdot 5 + 3$.

In order to work this problem, you need to follow the steps below.

Steps	Explanations of Steps
$1 - (3 + 7) \div 2 \cdot 5 + 3 = 1 - (10) \div 2 \cdot 5 + 3$	Always simplify everything in parentheses first.
$= 1 - 5 \cdot 5 + 3$	We have no exponents, but we do have multiplication and division. As we read the problem from left to right, the division comes first, and so we must do it next. Note that $10 \div 2 = 5$.
$= 1 - 25 + 3$	We must get rid of all the multiplication and division before we can move on to addition and subtraction. Note that $5 \cdot 5 = 25$.
$= -24 + 3$	The subtraction comes before the addition in the problem as we read the problem from left to right, and so we must do the subtraction next. Note that $1 - 25 = -24$.
$= -21$	The only thing left to do is add.

Whenever you have more than one operation to do inside a set of parentheses, you must follow the same order of operations to simplify whatever is inside the parentheses.

Example 2: Simplify $1 - (3 - 5 \bullet 2)$.

The steps we use to work this problem are described below.

$1 - (3 - 5 \bullet 2) = 1 - (3 - 10)$	We must simplify whatever is inside the parentheses first, and, inside these parentheses, we must do the multiplication before the subtraction.
$= 1 - (-7)$	Again, we must simplify expressions inside parentheses before moving on to anything else.
$= 8$	The only thing left to do is subtract.

Sometimes, you will see problems with parentheses inside of parentheses. In these problems, simplify the innermost parentheses first, and then move on to the next innermost set of parentheses. Also, to make the problems easier to understand, we will change some of the sets of parentheses to symbols that look like "{" and "}" or "[" and "]".

Example 3: Simplify $-5 + \{-7 - 3[1 - 45 \div (2 - 5)^2]\}^3$.

The steps we use to work this problem are described below.

$-5 + \{-7 - 3[1 - 45 \div (2 - 5)^2]\}^3$ $= -5 + \{-7 - 3[1 - 45 \div (-3)^2]\}^3$	We must work from the innermost set of parentheses outward. Note that $2 - 5 = -3$.
$= -5 + \{-7 - 3[1 - 45 \div 9]\}^3$	We must continue working from the innermost set of parentheses outward. Within the next innermost set of parentheses, the order of operations tells us that we must square the (-3) next.
$= -5 + \{-7 - 3[1 - 5]\}^3$	Within the innermost set of parentheses, we must multiply and divide before we can add or subtract. Note that $45 \div 9 = 5$.
$= -5 + \{-7 - 3[-4]\}^3$	Since the innermost set of parentheses now contains only one operation, we know we must now subtract. Note that $1 - 5 = -4$.
$= -5 + \{-7 - (-12)\}^3$	Now, within the next set of innermost parentheses, we must multiply before adding or subtracting. We chose to look at this as multiplying a positive 3 by -4, but you would get the same answer in the end if you looked at it as $-5 + \{-7 + (-3)[-4]\}^3$.
$= -5 + \{5\}^3$	We can now subtract. Note that $-7 - (-12) = 5$.
$= -5 + 125$	We must take care of the exponents before we can add or subtract.
$= 120$	The only thing left to do is add.

Example 4: Simplify $4|-11 + 4 \cdot 2|^2 - 3|15 - 1|$.

We will begin by stating that absolute value symbols are a lot like parentheses, and so we must simplify the expressions inside the absolute value symbols before we can move on to the other operations. This means that we can work this problem as shown below.

$4|-11 + 4 \cdot 2|^2 - 3|15 - 1|$ $= 4|-11 + 8|^2 - 3|15 - 1|$	We must begin by simplifying the expressions inside the absolute value symbols, and, within the absolute value symbols, we must use the order of operations. Therefore, we must begin by multiplying 4 • 2.
$= 4|-3|^2 - 3|14|$	Now, we can perform the addition and subtraction inside the absolute value symbols.
$= 4 \cdot 3^2 - 3 \cdot 14$	Next, we can take the absolute value of –3 and 14.
$= 4 \cdot 9 - 3 \cdot 14$	Since we have now gotten rid of the absolute value symbols, we can move on to exponents. Note that $3^2 = 9$.
$= 36 - 3 \cdot 14$	Next, we must multiply 4 • 9.
$= 36 - 42$	We must multiply 3 • 14 before we can move on to addition and subtraction.
$= -6$	The only thing left to do is subtract. Note that 36 – 42 = 36 + (–42), or –6.

Problems – Simplify each of the following.

131. 2 – 4 • 2 ÷ 2 = ________________

132. 24 ÷ 3 • 2 + 2 = _______________

133. 20 – (3 + 2 • 2) = ______________

134. 8 – (4 – 9) = __________________

135. 8 – 6 + 3 = ___________________

136. 7 • 3 – 15 ÷ 3 = _______________

137. 5 • 2 ÷ (4 – 2) = _______________

138. 16 ÷ 8 ÷ 2 = __________________

139. 1 – 24(–6 ÷ 2) = ______________

140. 2 – 3(8 – 14 ÷ 2 • 7) = __________

141. $5 + 1^2 =$ ____________________

142. $(3 + 1)^2 =$ ____________________

143. $1 - 3(1 + 1)^2 =$ ________________

144. $(-3)^2 - 4^3 + 5 =$ ______________

145. $5 - 2^4 =$ ____________________

146. $5 + (-2)^4 =$ __________________

147. $(-2)^4 =$ ______________________

148. $-2^4 =$ ______________________

Hint: Depending on how you look at it, the negative sign has to do with either addition/subtraction or multiplication/division. Therefore, only the 2 (and not the negative sign) is being raised to the fourth power.

149. $-5(2) + 4(-3)^3 =$ ______________

150. $7 + 18 \div (4 - 1)^2 =$ ____________

151. $1 - 28 \div (-2)^2 + 3 =$ ___________

152. $-6 + 3(2 - 5) =$ ________________

153. $-5^2 =$ _______________________

154. $-2^2 + 32 \div 2^3 =$ _______________

155. $3^{2+1} - 4(2) =$ ________________

156. $16 - [15 \div (2 + 1)] \bullet 4^2 =$ _______

157. $18 \div \{6 - [2(-3) + 3]\} =$ __________

163. $2|15 - 8| + |-3 + 5| =$ __________

158. $-3 - [9 + 18 \div 3 - (2 + 1)^2] =$ ______

164. $5 - 2|3 - 17(-2)| =$ ___________

159. $-7 + 6\{8 - [3 + 2(-5)]\} =$ __________

165. $-2|5 + (-9)(2)| + |3 - 5| =$ ______

160. $17 + 4\{3 + [1 - (4 - 1)]^2 - 5\} =$ _____

166. $5|7 - 4(2)|^2 - 7(3 - 5) =$ _______

161. $4 - \{3 + 5[4 - 5(2 + 3) + 5^2]\} =$ _____

167. $27(2 - 3)^4 \div |1 - 4|^2 =$ _________

162. $[2(3 + 4)]^0 - 2\{-8 + [2(0 - 1)^2]^3\} =$ ____

168. $|-24 - 4(-2)| \div (3 - 5)^3 =$ ______

Chapter Notes